Set Theory Cheat Sheet: Days 1-5

A **set** is a collection of distinct objects, called its elements.

- We write $x \in A$ to mean that x is an element of the set A.
- We write $A \subseteq B$ to mean that A is a subset of B. This means every element of A is also an element of B.

Permanent names for sets:

> $\emptyset$: The **empty set** is the unique set with no elements.
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> $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$: The **natural numbers**, **integers**, **rational numbers**, and **real numbers** respectively.

Operations on sets:

- Union: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.
- Intersection: $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.
- Cartesian product: $A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$.
- Power set: $\mathcal{P}(A) = \{X \mid X \subseteq A\}$.

Axioms for sets (ZFC):

> **Extensionality.** Two sets are the same if they have the same elements.
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> **Infinity.** There exists an infinite set.
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> **Pairing.** If A and B are sets, then $\{A, B\}$ is a set.
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> **Power set.** If A is a set, then $\mathcal{P}(A)$ is a set.
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> **Union.** The union of a family of sets is a set.
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> **Foundation.** Every non-empty set A has an element B which is disjoint from A.
>
> **Replacement.** If f is a function, then the range of f is a set.
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> **Separation.** If A is a set and P is a property, then $\{x \in A \mid P(x)\}$ is a set.
>
> **Choice.** The cartesian product of a non-empty family of nonempty sets is nonempty.

Two sets have the same **cardinality** if there is a one-to-one correspondence between them mapping elements of one set to elements of the other.
Permanent names for cardinalities:

- $\aleph_0$: “Countable infinity”. The cardinality of the set of natural numbers.
- $\mathfrak{c}$ or $2^{\aleph_0}$: “Continuum”. The cardinality of the set of real numbers.