



IHS Math Seminar project proposals: Logic and set theory



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You should join a logic project if...

- you liked Unit 1
- you enjoy thinking about what we really mean when we talk about mathematical objects
- infinity excites you
- you're interested in computer science / programming
- you enjoy meta jokes (including this one)

Project 0: Voting in Cantoria

Consider the country of Cantoria, home to infinitely many people. Cantoria is proudly a democracy, but has a problem: how do you run elections with an infinite population? With a nonprincipal ultrafilter, of course!

Goals:

- Use the language of set theory to create infinitary analogs of voting systems
- Understand ultrafilters and ultraproducts, as well as variations whose existence is independent of ZFC

Possible research question: Does ZFC prove the existence of an ultrafilter modeling an ideal Cantorian voting system?



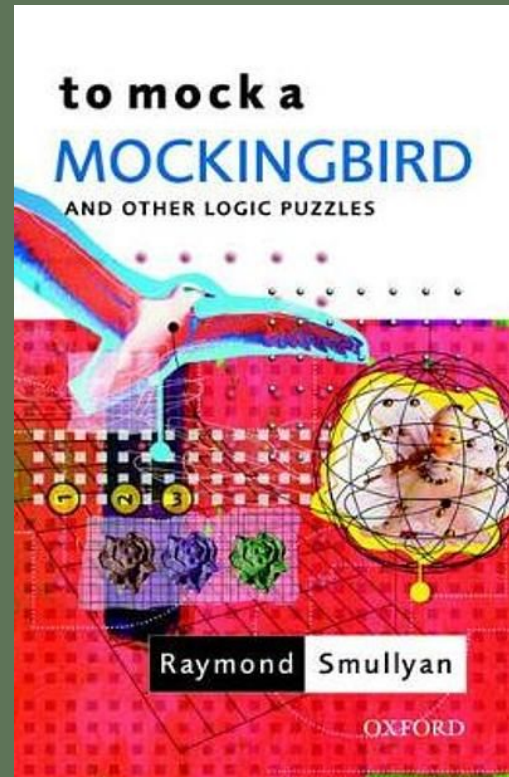
Project 1: Birdwatching and Combinatory Logic

"A certain enchanted forest is inhabited by talking birds. Given any birds A and B, if you call out the name of B to A, then A will respond by calling out the name of some bird to you; this bird we call AB."

Goals:

- Explore the world of combinatory logic using Raymond Smullyan's bird metaphor
- Discover the foundations of programming languages

Possible research question: Is every compositor a composition of blackbirds (B-combinators)?



Project 2: Programming Automata

Behold! I have constructed an automaton that plays Minecraft!

Are you sure? It looks like just a bunch of boxes and arrows...

Details, details. Yesterday I ran it for merely six hours and managed to punch a cow.

Goals:

- Learn about regular languages and the Myhill-Nerode theorem
- Ascend the Chomsky hierarchy: design DFAs, pushdown automata, and Turing machines for harder computational tasks

Possible research question: How many states are required for an automaton to compute the exponential function?

